2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
   (b) Find the absolute minimum value of f. Justify that your answer is an absolute minimum.
   (c) What is the range of f?
   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

GO ON TO THE NEXT PAGE

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1998 Calculus AB Scoring Guidelines

2. Let f be the function given by $f(x) = 2xe^{2x}$.
   (a) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.
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   (d) Consider the family of functions defined by $y = bxe^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of bxe^{bx} is the same for all nonzero values of b.

(a) $\lim_{x\to-\infty} 2xe^{2x} = 0$

$\lim_{x\to\infty} 2xe^{2x} = \infty$ or DNE

2 { 1: 0 as $x \to -\infty$; 1: ∞ or DNE as $x \to \infty$ }

(b) $f'(x) = 2e^{2x} + 2x \cdot 2 \cdot e^{2x} = 2e^{2x}(1 + 2x) = 0$

if $x = -1/2$

$f(-1/2) = -1/e$ or -0.368 or -0.367

$-1/e$ is an absolute minimum value because:

(i) $f'(x) < 0$ for all $x < -1/2$ and
$f'(x) > 0$ for all $x > -1/2$

–or–

(ii)

x		$-1/2$	
$f'(x)$	$-$	0	$+$

and $x = -1/2$ is the only critical number

3 { 1: solves $f'(x) = 0$; 1: evaluates f at student's critical point; 0/1 if not local minimum from student's derivative ; 1: justifies absolute minimum value; 0/1 for a local argument; 0/1 without explicit symbolic derivative }

Note: 0/3 if no absolute minimum based on student's derivative

(c) Range of $f = [-1/e, \infty)$
or $[-0.367, \infty)$
or $[-0.368, \infty)$

1: answer

Note: must include the left–hand endpoint; exclude the right–hand "endpoint"

(d) $y' = be^{bx} + b^2xe^{bx} = be^{bx}(1 + bx) = 0$

if $x = -1/b$

At $x = -1/b$, $y = -1/e$

y has an absolute minimum value of $-1/e$ for all nonzero b

3 { 1: sets $y' = be^{bx}(1 + bx) = 0$; 1: solves student's $y' = 0$; 1: evaluates y at a critical number and gets a value independent of b }

Note: 0/3 if only considering specific values of b

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AB 2 Board Note # 1

Part (d)

3/3 Argument with the following three ingredients:

1. The graph of $y = bxe^{bx}$ is a horizontal compression or expansion (with a reflection across the y–axis if $b < 0$) of the graph of $y = xe^x$.
2. The range of $y = bxe^{bx}$ is therefore the same as the range of $y = xe^x$.
3. Therefore the absolute minimum value of $y = bxe^{bx}$ is the same for all (non–zero) values of b.

0/3 Analyzing the horizontal compression/expansion of graphs of $y = bxe^{bx}$ for specific values of b.

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